The Cartesian Product of two Set

If we are given the Universal set U={1,2,3,4,5,6,7,8}, and sets A={3,4,5} and B={2,4,6,8} then we already know how to find A', namely A'={1,2,6,7,8}, B^c, namely B^c={1,3,4,7}, A∪B, namely A∪B={2,3,4,5,6,8}, and A∩B, namely A∩B={4}. We could even find A∪(B^c)={1,3,4,5,7} or (A^c)∩B={2,6,8}. Complement, union, and intersection are different, but the result is always a subset of the Universal set.

Now we are going to look at a new operator, the Cartesian Product. The result of the new operator is not a subset of the Universal set. Instead, the Cartesian Product of two sets produces something very different from those original sets. We use the symbol × to indicate the Cartesian Product. We read the × symbol as "cross". We define the Cartesian Product of two sets A and B as A×B={(g,h)|g∈A and h∈B}. In our case, using the sets A and B above, A×B={(3,2), (3,4), (3,6), (3,8), (4,2), (4,4), (4,6), (4,8), (5,2), (5,4), (5,6), (5,8)}. Thus, the result of taking the Cartesian Product is a new set, one that is made up of what we call "ordered pairs". The ordered pairs are created by mating, between parentheses, each element of the first set with each element of the second set, and separating those elements by a comma. Let us add a new set, D={2,5} to our discussion. Then, D×A={(2,3), (2,4), (2,5), (5,3), (5,4), (5,5)} and A×D={(3,2), (3,5), (4,2), (4,5), (5,2), (5,5)}. [Note that we created these sets in an orderly manner, but, as always, the actual order of the elements in a set is immaterial. We could rearrange the elements in any way without changing the set. However, the elements of these sets are ordered pairs. Although we could shuffle the ordered pairs we cannot shuffle the values within the ordered pair. That is, $(2,4) \in D \times A$ and we can put (2,4) anywhere in the list of $D \times A$ but we cannot make (2,4) into (4,2) in our listing of $D \times A$.]

Just looking at the two sets $D \times A$ and $A \times D$ we see that they are different. For example $(5,3) \in D \times A$ but $(5,3) \notin A \times D$, or $(4,2) \notin D \times A$ but $(4,2) \in A \times D$. Because $D \times A \neq A \times D$ we see that the Cartesian Product is <u>not commutative</u>.

One property of the Cartesian Product is that the cardinality of the Cartesian Product of two sets is the product of the cardinalities of the two sets. For any sets R and S we express this as $n(R \times S) = n(R) \bullet n(S)$. We can see that in our sets above where $n(A \times B) = 12$, $n(A) = 3$, and $n(B) = 4$. So, $n(A \times B) = n(A) \bullet n(B)$ becomes $12 = 3 \bullet 4$, and 3 times 4 is indeed 12.